A Proposal to Stabilize The Random Waypoint Mobility Model for Ad Hoc Network Simulation

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***Abstract*— It has been recently shown that the instantaneous average node speed for the random waypoint (RWP) mobility model may not reach a steady state regime due to velocity gradual decaying which can cause inaccurate results in simulations and communication protocol validations for mobile ad hoc networks (MANETs). This paper presents a modification to the RWP model, in which we propose to choose node speeds from a BETA(2,2) distribution, demonstrating analytically and by simulations that our idea stabilizes the instantaneous average node speed and consequently other important network metrics, like control overhead, number of dropped data packets and delivery delay. The proposal of alteration not only eliminates the decaying problem of the average node speed but also provides average values closer to the commonly supposed average velocity $\frac{V_{max}+V_{min}}{2}$ than those of the original RWP model. In addition, the proposed change for the RWP model can be readily incorporated into network simulators.**

***Index Terms*— Ad hoc networks, mobile networking, mobility, network modeling, simulation.**

I. Introduction

Wireless ad hoc networks require no base station and all the control and access tasks are distributed among nodes acting as peers [1]. That is, there is no network infrastructure, while nodes can be static or mobile. This makes such networks attractive in situations such as battle fields, catastrophe-relief efforts or environmental monitoring. Accordingly, communication protocols for ad hoc networks must be decentralized and utilize few resources, like information processing and energy. Such protocols must be tested under conditions reflecting various possible practical scenarios that a user may confront. In this context, the mobility effect on ad hoc networks has been investigated by many authors [2], [3], [4], [5], [6], [7]. These studies showed that the performance of mobile ad hoc networks is highly dependent on the mobility employed in simulations and its characteristics.

On the other hand, in order to be considered valid, the results from any network simulation must be obtained under steady state behavior, i.e., the convergency time shall be smaller than the total simulation interval. It implies that the initial transient is discarded for performance analysis. Therefore, mobility models that never attain a stationary regime must be avoided.

Among many mobility models used in the literature and in network simulators for MANETs, the random waypoint model is certainly the most employed [2], [3], [6], [7], [8], [9], [10], [11]. Its main features are the random choice of position and speed for the nodes, as well as the use of pause time between direction changes [2], [8]. In [6], Yoon *et al.* showed that the RWP model does not attain steady state regime under certain conditions. More specifically, they proved that the instantaneous average node speed consistently decreases over time for a given set of parameters which interferes with the network performance and therefore should not be directly used for simulations. Note that authors unaware of this problem have tested communication protocols under such conditions [3], [7], [8], [9], [10].

Previous works have proposed general frameworks to design mobility models with stable average node speed [12], [13], [14]; however, they did not present the impacts over networking performance metrics. In addition, the main features of the resultant models deviated from the principal characteristics of the RWP model, e.g., the average velocities of the used distributions for choosing node speed differ from the average value $\frac{V_{max}+V_{min}}{2}$ of the uniform distribution employed in RWP model, where V_{min} and V_{max} are the minimum and maximum velocities, respectively, that a node can select.

In this paper, we propose an alteration in the way the node speeds are chosen such that the RWP model always attain a steady state. We performed a study of the mobility showing analytically and by simulations how the use of a BETA(2,2) distribution [15] for node speed stabilizes this model for utilization in performance analysis of MANETs. The reason for choosing the BETA(2,2) function is because it does not change the RWP main features and this distribution can be readily incorporated into network simulators. Our intention is not to create a new mobility model, but to fix the severe decaying average speed problem of the widely used RWP model avoiding its damaging consequence over performance of ad hoc networks.

The remaining of this paper is organized as follows. Section II brings a review of the RWP model emphasizing its deficiency to attain a stationary regime if the speed parameters are not appropriately chosen. In Section III, we present a modification to the RWP model and show that if the function used to choose the velocity of the nodes is a BETA(2,2) distribution [15], then the instantaneous average node speed does not experience the stabilization problem. Section IV contains the performance results obtained in the JiST/SWANS network simulator [16] for the models studied. Finally, Section V concludes the paper.

II. Original RWP Mobility Model

A mobility model must resemble actual node movements in a network. The random waypoint mobility model try to approximate the reality by introducing pause time between changes in direction of the nodes, and it has been widely used to validate communication protocols in MANETs. Here we review the RWP model considering a rectangular network area with dimensions X_{max} x Y_{max}.

Assumption 1 *Original RWP mobility model [2], [6], [8]: Firstly, each node randomly chooses a initial position (x, y) in the network, where x and y are both uniformly distributed over $[0, X_{max}]$ and $[0, Y_{max}]$, respectively. Then, every node selects a destination (x', y') uniformly distributed in the network area and a speed v uniformly chosen from the range $[V_{min}, V_{max}]$, where V_{min} and V_{max} are the minimum and maximum velocities, respectively, that a node can choose, such that $0 < V_{min} < V_{max}$.[1] A node will then start travelling toward the (x', y') destination on a straight line using the chosen speed v. Upon reaching the selected destination, the node remains there for a pause time, either constant or randomly chosen from a given distribution. Upon expiration of the pause time, the next destination and speed are selected in the same way and the process repeats until the end of the simulation.*

For a mobility model, the instantaneous average node speed is defined by [6]

$$\overline{v}(t) = \frac{\sum_{i=1}^{N} v_i(t)}{N}, \tag{1}$$

where N is the total number of nodes and $v_i(t)$ is the speed of node i at time t. From Assumption 1, one may naively expect that $\overline{v}(t) = \frac{V_{max}+V_{min}}{2}$ $\forall\, t$ in the RWP model, regardless of V_{max} and V_{min}; however, it is not true. In [6], it was shown that a steady state cannot be attained in the cases where $V_{min} \rightarrow 0$, i.e., $\overline{v}(t)$ decay to zero over time such that $\lim_{t\rightarrow\infty} \overline{v}(t) = 0$. In practical terms, it means that if the range of $(0, V_{max}]$ is used for speed choices then the network simulation will never reach a stationary regime in terms of average node speed which may lead to inconsistent results when this model is employed to validate communication protocols in MANETs. An intuitive explanation for this fact is to observe that the RWP model selects destination and velocity for each node in a random and independent fashion where each node will maintain the chosen speed until it reaches the selected destination and then the process is repeated. During this procedure, the nodes that choose low speeds and long distances will remain trapped for a long time to these trips and depending on the total simulation period they may never reach their destinations. The nodes that select higher speeds and shorter distances will rapidly reach their destinations and soon they can choose new courses and velocities. As they repeat the procedure, these other nodes can choose low speeds and far destinations and they will also remain confined to slow journeys which dominate the average node speed, gradually taking the network to stagnation. Fig. 1 illustrates the behavior of the instantaneous average node speed for the original RWP model using the range of $(0, 20]$ meters per second (m/s) with zero pause time for a rectangular area with dimensions 1500m x 500m containing 50 mobile nodes. The presented curves were averaged over 30 distinct scenarios and the average speed is exhibited every 50 seconds where the simulations were performed in C++ [17]. At the beginning, i.e., at $t = 0$, $\overline{v}(t) = \frac{V_{max}+V_{min}}{2} = \overline{V}_{init}$, but $\overline{v}(t)$ consistently decreases with time. The curve for the range of $[1, 19]$ m/s is also shown in the figure for the original RWP model in which $\overline{v}(t)$ presents a initial decay due to the simulation transient and stabilizes near 6 m/s after 400 seconds. These results are in accordance with those presented in [6].

[1] Although the node speed is commonly assumed as uniformly distributed over $[0, V_{max}]$, it may result division by zero in the simulators [6].

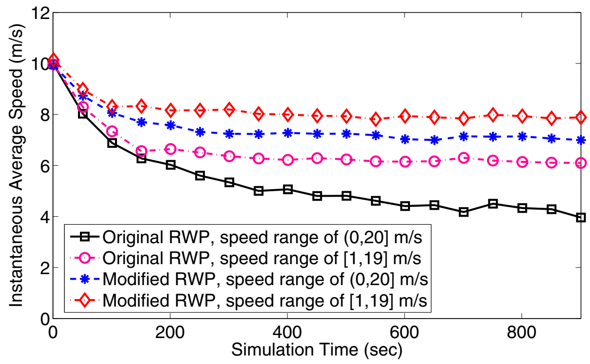

Fig. 1. Instantaneous average node speed for the original and modified RWP models employing ranges of $(0, 20]$ and $[1, 19]$ m/s, and zero pause time.

In [6], it was observed that pause times lead to fluctuations in the beginning of simulations; however, such effect is gradually reduced and the average node speed decaying is not consequence of pause time. Thus, we are going to assume zero pause time. Also, in the analytical study here presented, an unlimited and arbitrarily large area for the network is considered, instead of a rectangular area. These assumptions help to easily derive the results but they do not change the main conclusions.

Accordingly, the destination of a node is uniformly chosen from a circle of radius R_{max} centered at the current location of the node. Consequently, the probability density function (pdf) of the travel distance R in this model is given by [6]

$$f_R(r) = \tfrac{2r}{R_{max}^2}, \quad 0 \leq r \leq R_{max}, \tag{2}$$

and it has the following expectation

$$E[R] = \int_0^\infty r f_R(r) dr = \frac{2}{3} R_{max}. \tag{3}$$

The uniform pdf of the node speed V is

$$f_V(v) = \tfrac{1}{V_{max}-V_{min}}, \quad V_{min} \leq v \leq V_{max}, \tag{4}$$

where $0 < V_{min} < V_{max}$ and the mean of the random variable V is $E[V] = \int_0^\infty v f_V(v) dv = \frac{V_{max}+V_{min}}{2} = \overline{V}_{init}$. Note that $f_V(v)$ is indeed the BETA(1,1) distribution for the interval $[V_{min}, V_{max}]$ [15]. Moreover, $f_V(v)$ is symmetric in relation to the mean $E[V]$.

From Eqs. (2) and (4), it can be shown that the pdf of the travel time S is [6]

$$f_S(s) = \begin{cases} \frac{2s}{3R_{max}^2}(V_{max}^2 + V_{min}^2 + V_{max}V_{min}) & 0 \leq s \leq \frac{R_{max}}{V_{max}} \\ \frac{2R_{max}}{3(V_{max}-V_{min})s^2} - \frac{2sV_{min}^3}{3R_{max}^2(V_{max}-V_{min})} & \frac{R_{max}}{V_{max}} \leq s \leq \frac{R_{max}}{V_{min}} \\ 0 & s \geq \frac{R_{max}}{V_{min}}, \end{cases}$$

and the expected travel time is obtained by

$$E[S] = \int_0^\infty s\, f_S(s) ds = \tfrac{2R_{max}}{3(V_{max}-V_{min})} \ln\left(\tfrac{V_{max}}{V_{min}}\right). \tag{5}$$

In [6], it was shown that the time average of the speed $(\overline{V})$ for a given node can be obtained from $v(t)$ such that

$$\overline{V} = \lim_{T\rightarrow\infty} \frac{1}{T} \int_0^T v(t) dt = \frac{E[R]}{E[S]}, \tag{6}$$

which can be taken as the steady state expected node speed, assuming that the ensemble average equals time average as $t \to \infty$. Substituting Eqs. (3) and (5) in Eq. (6) gives the time average of the speed in the original RWP model ($\overline{V}_{orig}$) [6], i.e.,

$$\overline{V}_{orig} = (V_{max} - V_{min}) / \ln(V_{max}/V_{min}). \qquad (7)$$

From Eqs. (7) and (5), $\overline{V}_{orig} \to 0$ as $V_{min} \to 0$, because $E[S] \stackrel{V_{min}\to 0}{\longrightarrow} \infty$. If $V_{min} > 0$ then a steady non-null average speed is reached. However, to quickly attain a stationary regime, one has to set $V_{min} >> 0$ because the smaller V_{min}, the longer the decay period until the steady state is achieved (see Fig. 1).

III. Modified RWP Mobility Model

The solution of setting $V_{min} >> 0$ to avoid the consistent speed decay restricts the use of the RWP model, e.g., preventing its utilization in testing scenarios where node velocity can be very low. In this section, we describe a modification that eliminates the decaying problem of the instantaneous average speed rendering the RWP model stable. We propose to employ a BETA(2,2) distribution [15] for choosing node speeds instead of the uniform distribution presented in Eq. (4). We show that, in this case, the instantaneous average node speed does not decay to zero as $V_{min} \to 0$ and quickly attains a steady state.

The proposed distribution for the node speed V in the modified RWP model is the pdf

$$f_V(v) = -\frac{6\left(v - \frac{V_{max}+V_{min}}{2}\right)^2}{(V_{max}-V_{min})^3} + \frac{3}{2(V_{max}-V_{min})}, \qquad (8)$$

for $V_{min} \leq v \leq V_{max}$, where $0 < V_{min} < V_{max}$. This is the BETA(2,2) pdf for the interval $[V_{min}, V_{max}]$ [15]. $f_V(v)$ is symmetric in relation to the mean $E[V]$ which equals $\frac{V_{max}+V_{min}}{2} = \overline{V}_{init}$, analogous to the uniform distribution of the original RWP model. In addition, the BETA(2,2) distribution can be readily implemented into network simulators because this pdf is available in common programming languages (see GSL [18] or SSJ [19] libraries, for example).

Assumption 2 *Modified RWP mobility model: (i) each node randomly chooses a initial position (x, y) in the network, where x and y are both uniformly distributed over $[0, X_{max}]$ and $[0, Y_{max}]$, respectively. (ii) Then, every node selects a destination (x', y') uniformly distributed in the network area and a speed v according to the BETA(2,2) distribution given in Eq. (8). (iii) A node will then start travelling toward the (x', y') destination on a straight line using the chosen velocity v. (iv) Upon reaching the selected destination, the next destination and speed are chosen in the same way as in (ii) and the process repeats until the end of the simulation.* [2]

Assumption 2 does not change other RWP mobility features, like node distribution in the network area and average node neighbor percentage, [3] because they are consequence of the random procedure for choosing position and velocity of the nodes which is essentially the same as in Assumption 1, except for the speed pdf.

[2]As mentioned before, we are considering zero pause time because it does not change our main conclusions and facilitates our analysis.

[3]The average node neighbor percentage is the cumulative percentage of total nodes that are a given node's neighbor. For example, if there are 200 nodes in the network and a node has 20 neighbors, then the nodes current neighbor percentage is 10% or 0.1 [2].

Fig. 2 shows the spatial distribution for the mobile nodes after 900 seconds of simulation in a rectangular area with dimensions 1500m x 500m containing 200 mobile nodes for the modified RWP model with speed range of (0,20] m/s. Analogous to the original RWP model, the node distribution has a higher concentration near the center of the network [20].

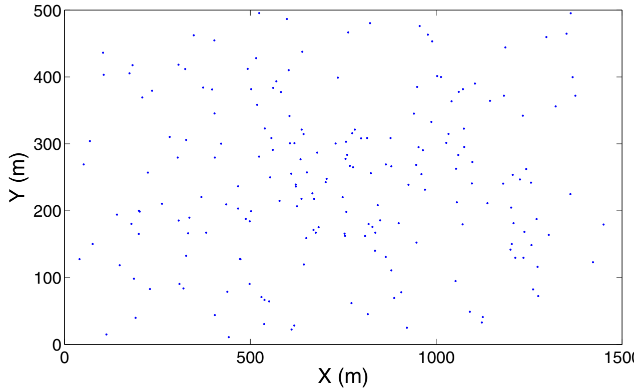

Fig. 2. Spatial node distribution after 900 seconds of simulation for the modified RWP mobility model in a rectangular network area with dimensions 1500m x 500m containing 200 mobile nodes, speed range of (0,20] m/s.

Fig. 3 illustrates the evolution of the average node neighbor percentage [2] as function of simulation time which shows that the original and modified RWP models have reasonable similar behavior.

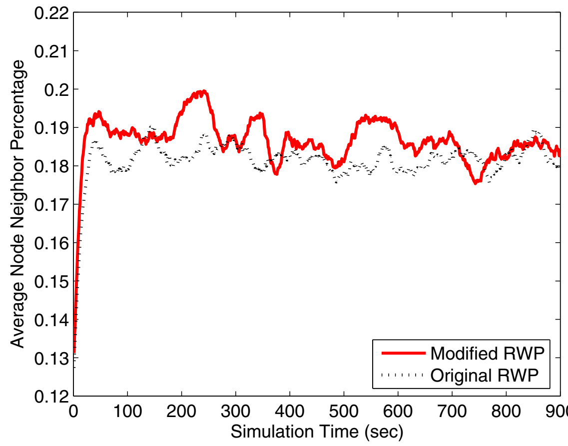

Fig. 3. Average node neighbor percentage as a function of simulation time for the original and modified RWP mobility models in a rectangular network area with dimensions 1500m x 500m containing 200 mobile nodes, speed range of (0,20] m/s.

The following theorem is obtained using Assumption 2.

Theorem 1 *The pdf of the travel time S for the modified RWP mobility model is given by*

$$f_S(s) = \begin{cases} \frac{2Ks}{R_{max}^2(V_{max}-V_{min})^3} & 0 \leq s \leq \frac{R_{max}}{V_{max}} \\ -\frac{12R_{max}^3}{5(V_{max}-V_{min})^3 s^4} + \frac{3(V_{max}+V_{min})R_{max}^2}{(V_{max}-V_{min})^3 s^3} & \frac{R_{max}}{V_{max}} \leq s \leq \frac{R_{max}}{V_{min}} \\ \quad -\frac{4V_{max}V_{min}R_{max}}{(V_{max}-V_{min})^3 s^2} + \frac{V_{min}^4(V_{max}-\frac{3}{5}V_{min})s}{R_{max}^2(V_{max}-V_{min})^3} & \\ 0 & s \geq \frac{R_{max}}{V_{min}}, \end{cases} \qquad (9)$$

where K is the following constant

$$K = -\frac{6}{5}(V_{max}^5 - V_{min}^5) + \frac{3}{2}(V_{max}+V_{min})(V_{max}^4 - V_{min}^4) -2V_{max}V_{min}(V_{max}^3 - V_{min}^3), \quad (10)$$

and the expected travel time is

$$E[S]=\frac{R_{max}}{(V_{max}-V_{min})^3}\left[\frac{2K}{3V_{max}^3}+\frac{9(V_{max}^2-V_{min}^2)}{5}-4V_{max}V_{min}\ln\left(\frac{V_{max}}{V_{min}}\right)+\frac{(V_{max}-\frac{3}{5}V_{min})(V_{min}-\frac{V_{min}^4}{V_{max}^3})}{3}\right]. \quad (11)$$

Proof: See Appendix.

From Eqs. (3), (6) and (11), we obtain the time average of the speed in the modified RWP model ($\overline{V}_{modif}$) as follows

$$\overline{V}_{modif} = E[R]/E[S] = 2(V_{max}-V_{min})^3\left[\frac{2K}{V_{max}^3} - 12V_{max}V_{min}\ln\left(\frac{V_{max}}{V_{min}}\right) + \frac{27}{5}(V_{max}^2-V_{min}^2)+(V_{max}-\frac{3}{5}V_{min})(V_{min}-\frac{V_{min}^4}{V_{max}^3})\right]^{-1}. \quad (12)$$

It is straightforward to verify that

$$\lim_{V_{min}\to 0}\overline{V}_{modif} = \frac{V_{max}}{3}. \quad (13)$$

That is, if we employ a BETA(2,2) distribution for speed choice in the RWP model, instead of the uniform distribution, then the time average of the node speed does not decay to zero when $V_{min} \to 0$ and it stabilizes at $\frac{V_{max}}{3}$. Therefore, our proposal of modification eliminates the decaying problem presented by the original RWP model allowing the average node speed to attain steady state.

Similar to the original RWP model, we simulated in C++ (with the GSL library [18]) a rectangular area with dimensions 1500m x 500m containing 50 mobile nodes for the modified RWP model in which the results were averaged over 30 distinct scenarios and they are shown in Fig. 1. We see that the average speed attains steady state after 400 seconds of simulation (after the transient) using the ranges of [1,19] or (0,20] m/s which agrees with the analytical results given by Eqs. (12) and (13).

Table I presents a list of speed ranges and respective speed averages. $\overline{V}_{sim}$ is the time average of the speed from C++ simulation for the modified RWP model employing a 1500m x 500m area containing 50 mobile nodes for a total simulation time of 1000 seconds in which the first 500 seconds were discarded due to the initial transient. The simulation results were averaged over 10 different scenarios. Note that $\overline{V}_{init}$ was defined as $\frac{V_{max}+V_{min}}{2}$, $\overline{V}_{orig}$ is given by Eq. (7), and $\overline{V}_{modif}$ is obtained from Eq. (12). We observe agreement among the simulation ($\overline{V}_{sim}$) and analytical ($\overline{V}_{modif}$) results. We also see that $\overline{V}_{modif} > \overline{V}_{orig}$ which indicates that our proposal of modification not only eliminates the decaying problem of the average speed, but it also provides average values closer to $\overline{V}_{init}$ than those of the original RWP model.

IV. Networking Performance Results

In order to show the importance of employing a stable mobility model in performance analysis of MANETs, we performed simulations for the original and modified RWP models utilizing the JiST/SWANS network simulator [16] with the SSJ library [19]. The results relative to speed are similar to those obtained in C++ (presented in Fig. 1 and Table I). The simulation environment was a rectangular area network with dimensions 1500m x 500m containing 50 mobile nodes. The results were averaged over 30 distinct scenarios and were calculated at every 100 seconds, generating 9 points different from zero in 900 seconds of simulation for each presented curve. Similar to [6], the data traffic was constant bit rate (CBR) in which 30 source nodes, randomly chosen at each run, transmitted packets of length 64 bytes at a rate of 4 packets per second. For routing, we used ad hoc on-demand distance vector algorithm (AODV) [11]. Here, only the following three metrics were investigated because our intention is not to provide an exhaustive performance comparison, but to show how our proposal stabilizes important network measures related to control overhead, discarded data packets and delivery delay.

TABLE I

$\overline{V}_{init}$, $\overline{V}_{orig}$, $\overline{V}_{modif}$ AND $\overline{V}_{sim}$ FOR VARIOUS SPEED RANGE (IN m/s).

Speed range	$\overline{V}_{init}$	$\overline{V}_{orig}$	$\overline{V}_{modif}$	$\overline{V}_{sim}$
(0,20]	10	0	6.67	6.92
[1,19]	10	6.11	7.84	7.86
[2,18]	10	7.28	8.44	8.50
[3,17]	10	8.07	8.87	8.88
[4,16]	10	8.66	9.21	9.20
[5,15]	10	9.10	9.47	9.46
[6,14]	10	9.44	9.67	9.66
[7,13]	10	9.69	9.82	9.79
[8,12]	10	9.87	9.91	9.92
[9,11]	10	9.97	9.98	9.97
[1,21]	11	6.57	8.54	8.62
[2,22]	12	8.34	9.91	9.92
[3,23]	13	9.82	11.16	11.19
[4,24]	14	11.16	12.35	12.37
[5,25]	15	12.43	13.49	13.49
[6,26]	16	13.64	14.61	14.72
[7,27]	17	14.82	15.71	15.73
[8,28]	18	15.96	16.80	16.82
[9,29]	19	17.09	17.87	17.91
[10,30]	20	18.20	18.94	19.01

- *Number of routing overhead packets:* This measure considers all control packets generated by the AODV routing protocol to discover and maintain routes. Fig. 4 illustrates the effect of the velocity consistent decay caused over this metric by the original RWP mobility model employing the speed range of (0,20] m/s, while this performance measure was stabilized in the modified RWP model.

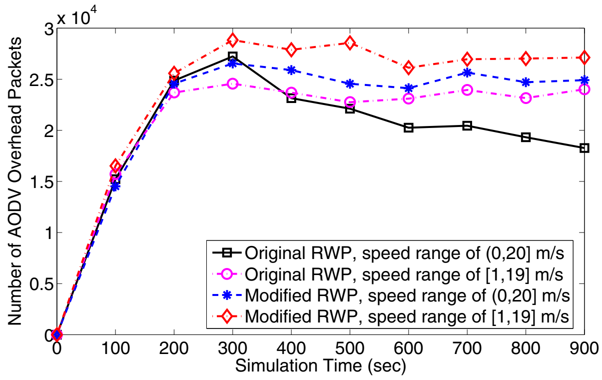

Fig. 4. Number of routing overhead packets as a function of simulation time.

- *Number of dropped data packets:* This performance measure, illustrated in Fig. 5, quantifies the amount of transmitted data packets discarded by routers in the path to destinations due to errors occurred in the physical or upper layers. It is also clear the influence from the average speed decaying over the curve related to the range of (0,20] m/s in the original RWP model, whereas this metric was stabilized in the modified RWP model.

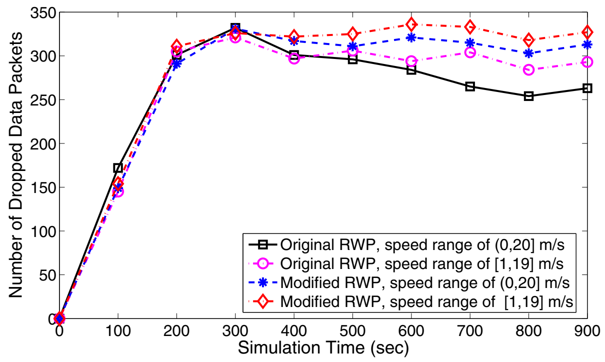

Fig. 5. Number of dropped data packets as a function of simulation time.

- *Data packet delivery delay:* It measures the time elapsed between transmission and successful delivery of a data packet from source to destination. This measure is averaged over all counted packets. Fig. 6 shows the shortest delay for the case of the original RWP model employing range of (0,20] m/s because the average speed decaying causes the network to become more static with time resulting in less changes on route discovery and maintenance.

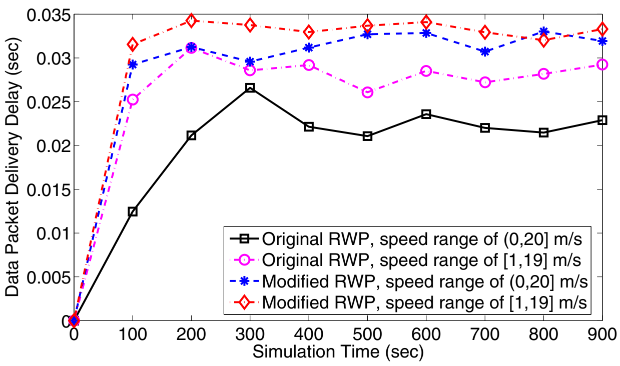

Fig. 6. Data packet delivery delay as a function of simulation time.

Note that, in all cases, the results showed for the original RWP model are in agreement with those presented in [6]. In addition, our proposal of modification to the RWP model stabilized the investigated performance measures. The results also confirm that important network metrics are related to instantaneous average node speed.

V. Conclusion

This paper proposed and analyzed a modification that stabilizes the random waypoint mobility model used to evaluate performance of MANETs. We showed that the use of a BETA(2,2) distribution for choosing node speed, instead of the uniform distribution, avoids the gradual decaying with time of the instantaneous average node speed when the minimum velocity in the choice range is set to zero. Analytical and simulation results were presented. Beyond eliminating the decaying problem of the instantaneous average node speed, our proposal of modification also provides average values closer to the commonly supposed average velocity $\frac{V_{max}+V_{min}}{2}$ than those of the original RWP model. In addition, the alteration allowed the investigated network performance metrics to attain steady state and it corroborates the importance of using a stable mobility model when communication protocols are under evaluation in ad hoc networks. Future work consists of studying other probability distributions for node speed in the RWP model and their effects over network performance.

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Appendix

A. Proof of Theorem 1

As in Section II, we assume an unlimited and arbitrarily large area for the network, instead of a rectangular area. In order to calculate the pdf of the travel time S, we first obtain its probability distribution $P(S \leq s)$ from Fig. 7, considering three possible cases [6].

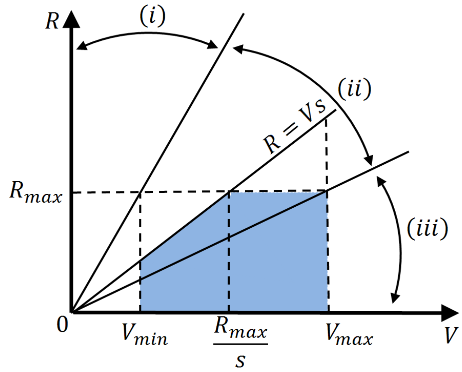

Fig. 7. Distance-Speed graph. (i), (ii) and (iii) are the three regions (cases) of interest.

In addition, from Assumption 2, we use that the random variables R and V are independent and that their pdfs are given by Eqs. (2) and (8), respectively.

(i) For the case $s \geq \frac{R_{max}}{V_{min}}$ (i.e., $R_{max} \leq V_{min}s$), we have

$$P(S \leq s) = \int_{V_{min}}^{V_{max}} \int_{0}^{R_{max}} f_{R,V}(r,v)drdv = 1.$$

(ii) For the case $\frac{R_{max}}{V_{max}} \leq s \leq \frac{R_{max}}{V_{min}}$ (i.e., $V_{min}s \leq R_{max} \leq V_{max}s$), we obtain

$$P(S \leq s) = \underbrace{\int_{V_{min}}^{\frac{R_{max}}{s}} \int_{0}^{vs} f_{R,V}(r,v)drdv}_{I_1} + \underbrace{\int_{\frac{R_{max}}{s}}^{V_{max}} \int_{0}^{R_{max}} f_{R,V}(r,v)drdv}_{I_2}.$$

Solving for I_1, it follows that

$$I_1=\int_{V_{min}}^{\frac{R_{max}}{s}} f_V(v)\int_0^{vs} \frac{2r}{R_{max}^2}drdv=\frac{s^2}{R_{max}^2}\int_{V_{min}}^{\frac{R_{max}s}{s}} v^2 f_V(v)dv$$
$$=\frac{s^2}{R_{max}^2}\left[-\int_{V_{min}}^{\frac{R_{max}}{s}} \frac{6v^2}{(V_{max}-V_{min})^3}\left(v-\frac{V_{max}+V_{min}}{2}\right)^2 dv\right.$$
$$\left.+\frac{3}{2(V_{max}-V_{min})}\int_{V_{min}}^{\frac{R_{max}}{s}} v^2 dv\right]$$
$$=\frac{1}{(V_{max}-V_{min})^3}\left[-\frac{6R_{max}^3}{5s^3}+\frac{3(V_{max}+V_{min})R_{max}^2}{2s^2}\right.$$
$$\left.-\frac{2V_{max}V_{min}R_{max}}{s}+\frac{V_{min}^4(V_{max}-\frac{3}{5}V_{min})s^2}{2R_{max}^2}\right].$$

Analogously, for I_2, it results that

$$I_2=\int_{\frac{R_{max}}{s}}^{V_{max}} f_V(v)\int_0^{R_{max}} \frac{2r}{R_{max}^2}drdv=\int_{\frac{R_{max}}{s}}^{V_{max}} f_V(v)dv$$
$$=\frac{1}{(V_{max}-V_{min})^3}\left[\frac{2R_{max}^3}{s^3}-\frac{3(V_{max}+V_{min})R_{max}^2}{s^2}-2V_{max}^3\right.$$
$$\left.+\frac{6V_{max}V_{min}R_{max}}{s}-6V_{max}^2V_{min}+3(V_{max}+V_{min})V_{max}^2\right].$$

By adding I_1 and I_2 with rearrangement of terms, we arrive at

$$P(S\le s)=\frac{1}{(V_{max}-V_{min})^3}\left[\frac{4R_{max}^3}{5s^3}-\frac{3(V_{max}+V_{min})R_{max}^2}{2s^2}+\right.$$
$$\frac{4V_{max}V_{min}R_{max}}{s}+\frac{V_{min}^4(V_{max}-\frac{3}{5}V_{min})s^2}{2R_{max}^2}-$$
$$\left.2V_{max}^3-6V_{max}^2V_{min}+3\left(V_{max}+V_{min}\right)V_{max}^2\right].$$

(iii) For the case $0\le s\le\frac{R_{max}}{V_{max}}$ (i.e., $0\le sV_{max}\le R_{max}$), we have

$$P(S\le s)=\int_{V_{min}}^{V_{max}}\int_0^{vs} f_{R,V}(r,v)drdv=\frac{s^2}{R_{max}^2}\int_{V_{min}}^{V_{max}}v^2 f_V(v)dv$$
$$=\frac{s^2}{R_{max}^2(V_{max}-V_{min})^3}\left[-\frac{6}{5}(V_{max}^5-V_{min}^5)+\right.$$
$$\frac{3}{2}(V_{max}+V_{min})(V_{max}^4-V_{min}^4)-$$
$$\left.2V_{max}V_{min}(V_{max}^3-V_{min}^3)\right]$$
$$=\frac{Ks^2}{R_{max}^2(V_{max}-V_{min})^3},$$

where $K=-\frac{6}{5}(V_{max}^5-V_{min}^5)+\frac{3}{2}(V_{max}+V_{min})(V_{max}^4-V_{min}^4)-2V_{max}V_{min}(V_{max}^3-V_{min}^3)$.

The pdf of travel time S is obtained by differentiating $P(S\le s)$ with respect to s, which results

$$f_S(s)=\frac{dP(S\le s)}{ds}=$$
$$\begin{cases}\frac{2Ks}{R_{max}^2(V_{max}-V_{min})^3} & 0\le s\le\frac{R_{max}}{V_{max}}\\ -\frac{12R_{max}^3}{5(V_{max}-V_{min})^3s^4}+\frac{3(V_{max}+V_{min})R_{max}^2}{(V_{max}-V_{min})^3s^3} & \frac{R_{max}}{V_{max}}\le s\le\frac{R_{max}}{V_{min}}\\ -\frac{4V_{max}V_{min}R_{max}}{(V_{max}-V_{min})^3s^2}+\frac{V_{min}^4(V_{max}-\frac{3}{5}V_{min})s}{R_{max}^2(V_{max}-V_{min})^3} & \\ 0 & s\ge\frac{R_{max}}{V_{min}}.\end{cases}$$

From the above pdf, the expected travel time is

$$E[S]=\int_0^\infty sf_S(s)ds=\int_0^{\frac{R_{max}}{V_{max}}}\frac{2Ks^2}{R_{max}^2(V_{max}-V_{min})^3}ds+$$
$$\int_{\frac{R_{max}}{V_{max}}}^{\frac{R_{max}}{V_{min}}}\left[-\frac{12R_{max}^3}{5(V_{max}-V_{min})^3s^3}+\frac{3(V_{max}+V_{min})R_{max}^2}{(V_{max}-V_{min})^3s^2}-\right.$$
$$\left.\frac{4V_{max}V_{min}R_{max}}{(V_{max}-V_{min})^3s}+\frac{V_{min}^4(V_{max}-\frac{3}{5}V_{min})s^2}{R_{max}^2(V_{max}-V_{min})^3}\right]ds$$
$$=\frac{R_{max}}{(V_{max}-V_{min})^3}\left[\frac{2K}{3V_{max}^3}+\frac{9(V_{max}^2-V_{min}^2)}{5}-\right.$$
$$\left.4V_{max}V_{min}\ln\left(\frac{V_{max}}{V_{min}}\right)+\frac{(V_{max}-\frac{3}{5}V_{min})(V_{min}-\frac{V_{min}^4}{V_{max}^3})}{3}\right],$$

which finishes the proof. ■

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